

Analysis and Throughput Optimization of Selective Chase Combining for OFDM Systems

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Abstract

In this paper, we present throughput analysis and optimization of bandwidth efficient selective retransmission method at modulation layer for conventional Chase Combining (CC) method under orthogonal frequency division multiplexing (OFDM) signaling. Most of the times, there are fewer errors in a failed packet and receiver can recover from errors receiving *partial* copy of original frame. The proposed selective retransmission method at modulation layer for OFDM modulation requests retransmission of information corresponding to the poor quality subcarriers. In this work, we propose cross-layer multiple selective Chase combining (MSCC) method and Chase combining with selective retransmission (CCWS) at modulation level. We also present bit-error rate (BER) and throughput analysis of the proposed MSCC and CCWS methods. In order to maximize throughput of the proposed methods under OFDM signaling, we formulate optimization problem with respect to amount of information to be retransmitted in selective retransmission in the event of packet failure. We present tight BER upper bounds and tight throughput lower bounds for the proposed selective Chase combining methods. The simulation results demonstrate significant throughput gain of the optimized selective retransmission methods over conventional retransmission methods. The throughput gain of the proposed selective retransmission at modulation layer are also holds for conventional for hybrid automatic repeat request (HARQ) methods.

I. INTRODUCTION

The contemporary wireless communication standards such as LTE-advanced [1] integrate new technologies to meet increasing need of high data rate. The current and future communication systems employ multiple-input multiple-output (MIMO) technologies due to their potential to achieve higher data rate and diversity. In order to assure error-free communication with high throughput, packet error detection and correction protocols have evolved over time [2]. The

automatic repeat reQuest (ARQ) method guarantees error free data transfer using cyclic redundancy check (CRC) approach. The concept of hybrid ARQ (HARQ) integrates ARQ and forward error correction (FEC) codes [2], [3] to provide effective means of enhancing overall throughput of communication systems. In the event of packet failure, advanced form of HARQ incorporates joint decoding by combining soft information from multiple transmissions of a failed packet. Thus, HARQ is one of the most important technologies embedded in the latest communication systems such as high-speed down link packet access (HSDPA), universal mobile telecommunications system (UMTS) that pervade 3G and 4G wireless networks to ensure data reliability.

In type-I HARQ, receiver request retransmission of a failed packet and discards observation of the failed packet. Type-II HARQ is most commonly used method and achieves higher throughput. The type-II HARQ is divided into Chase combining HARQ (CC-HARQ) and incremental redundancy HARQ (IR-HARQ). In CC-HARQ, receiver preserves observations of the failed packet and request retransmission of full packet. The receiver Chase combines [4] observations of the failed packet and retransmitted packet to decode packet. In the event of packet failure under IR-HARQ, receiver request retransmission of more parity information to recover from error. In response to retransmission request, transmitter sends more parity bits lowering code rate of FEC. The receiver combines new parity bits with buffer observation for FEC decoding.

Most of the research conducted on HARQ focused on ARQ and FEC [3], [5] without exploring modulation layer. Throughput of capacity achieving codes FECs such as LDPC and Turbo codes is optimized for Rayleigh fading channel in [6] with ARQ and HARQ protocols. Mutual information based performance analysis for HARQ over Rayleigh fading channel is provided in [7]. Optimal power allocation for Chase combining based HARQ is optimized in [8]–[12]. Without exploiting channel state information and frequency diversity of the frequency selective channel, partial retransmission of the original symbol stream of a failed packet is addressed in [13]–[15]. These methods retransmit punctured packet in predetermined fashion. Furthermore, the complexity of joint detection for *partial* retransmission is [13], [14], [16] not tractable. Partial retransmission of orthogonal space-time block (OSTB) coded [17] OFDM signaling is proposed in [15]. In [18], for conventional ARQ protocol, full packet retransmission at modulation layer is employed when channel gain is below a threshold value without buffering observations of low SNR channel realization. In a typical failed packet, there are small number

of corrupted bits and retransmission of full packet is not necessary. The receiver can recover from errors by retransmission of potentially culprit bits. The OFDM signaling allows to identify poor quality bits corresponding to the subcarriers that have low signal-to-noise ratio (SNR). Selective retransmission at modulation layer of OFDM signaling proposed in [19]–[22] achieves throughput gain as compared to conventional HARQ methods. Throughput optimization of selective retransmission and performance analysis is not addressed in [19].

The motivation of selective retransmission owing to the fact that in the event of failed packet under OFDM signaling at MAC layer, often receiver can recover from error by retransmitting *partial* information corresponding to the poor quality subcarriers. An OFDM signaling allows retransmission of information transmitted over poor quality subcarrier at physical layer (PHY) selectively. After receiving copy of information symbols corresponding to poor quality subcarriers, receiver jointly decode data in Chase combining fashion. In this work, we propose low complexity and bandwidth efficient multiple selective Chase combining (MSCC) and Chase combining with selective retransmission (CCWS) methods in cross-layer fashion at modulation layer for OFDM signaling. We also provide bit-error rate (BER) and throughput analysis in terms tight upper BER bound and lower throughput bound, respectively for the proposed retransmission schemes. The amount of information to be retransmitted for each subcarrier in the event of failed packet is function of signal-to-noise ratio (SNR) of the corresponding subcarrier. Using norm of channel gain for each subcarriers as channel quality measure, we also optimized threshold τ on channel norm for selective retransmission in order to maximize throughput. The simulation results demonstrate that proposed methods offers substantial throughput gain over conventional CC method in low SNR regime. Our results also show that there is marginal gap between analytical bounds and simulation results (Monte Carlo method) for BER and throughput of the proposed methods. The throughput gain of the proposed schemes also hold with FEC.

We organize our manuscript as follows. First, we present system model and problem formulation (MSCC and CCWS) for selective Chase combining of OFDM system in Section II-A and Section II-B. In Section III, we present BER analysis of MSCC for one selective retransmission at modulation layer and CCWS methods. Throughput analysis for SCC and CCWS are presented in Section IV. Throughput Optimization is performed in Section V. We discuss our results in Section VI. Finally, we conclude the purposed work in Section VII.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider OFDM system for selective retransmission with n_r receive antennas. Transmitter transmits information symbol using OFDM signaling over frequency selective channel of L coefficients. An OFDM signaling converts frequency selective channel \mathbf{h}_i , where $i = 1, \dots, n_r$, from transmit antenna to the receive antenna i into N_s parallel channel [23]. Thus, a channel gain vector of ℓ -th subcarrier between transmitter and receiver pair is $H(\ell) = [H_1^T(\ell) \cdots H_{n_r}^T(\ell)]^T$, where elements of channel vector $H(\ell)$ are generated by applying Fourier transformation matrix $F \in \mathcal{C}^{N_s \times N_s}$ on frequency selective channel \mathbf{h}_i . The elements of vector $H(\ell)$ are independent and identically distributed (i.i.d.) with distribution $\mathcal{N}_c(0, 1)$ [23]. Note that for $n_r = 1$, SIMO-OFDM system becomes SISO-OFDM system. The matrix model of the received vector $\mathbf{y}(\ell)$ for the ℓ -th subcarrier can be written as

$$\mathbf{y}(\ell) = H(\ell) \cdot s(\ell) + \mathbf{w}(\ell), \quad (1)$$

where vector $\mathbf{w}(\ell) \sim \mathcal{N}_c(0, N_0 I)$ is an additive white Gaussian noise vector. A typical failed packet has few erroneous bits and bits transmitted over OFDM subcarriers with small channel norm $\|H(\ell)\|^2$ are more susceptible to the impairments. OFDM signaling allows retransmission of targeted information symbols corresponding to the poor quality subcarriers instead of unnecessary retransmission of full packet [19]. The joint detection by combining observation of the first transmission and subsequent selective retransmission of the poor quality subcarriers enhances throughput of the transceiver under OFDM signaling. The selective retransmitted information improves bit-error rate of the poor quality subcarriers resulting into throughput gain. Note that retransmission of more information does not increase throughput linearly. The amount of information to be retransmitted is controlled by threshold τ on the channel norm $\|H(\ell)\|^2$ of the ℓ -th subcarriers. The optimization of threshold τ in order to maximize throughput η of the transceiver under selective retransmission is the main focus of this work. Throughput of selective retransmission is function of probability of error, which in fact is function of τ . We present uncoded BER analysis and throughput analysis in Section III and IV, respectively, in order to optimize parameter τ in Section V. Next, we provides description of proposed MSCC and CCWS methods.

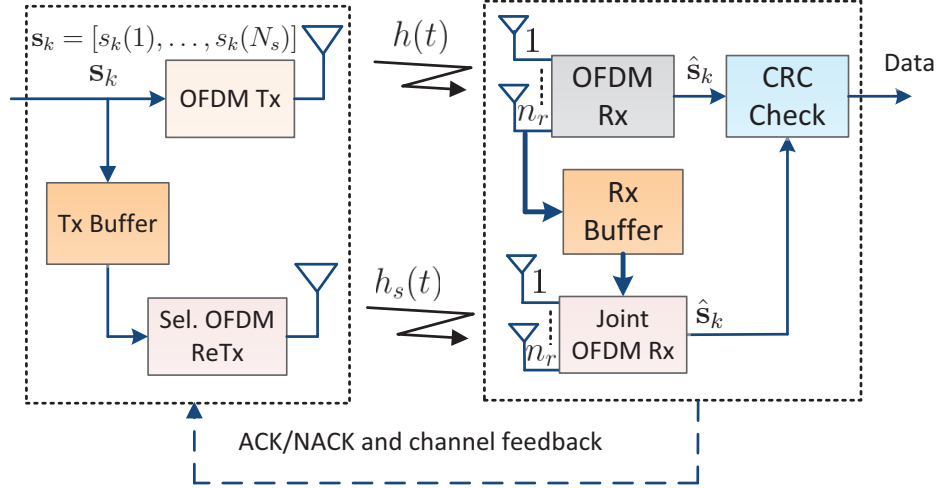


Fig. 1. Selective retransmission model for SIMO-OFDM system at modulation layer

B. Problem Formulation

Now we present our proposed cross-layer MSCC and CCWS methods for OFDM signaling. In both MSCC and CCWS methods, medium access control (MAC) layer initiates retransmission in the event of CRC failure. Based on quality of the subcarriers, instead of full retransmission under conventional Chase combining, PHY layer initiates selective retransmission of information symbols corresponding to the poor quality subcarriers along with NACK signal. We use norm of subcarrier gain vector $\|H(\ell)\|^2$ as channel quality measure. Note that OFDM signaling allows retransmission of information symbols transmitted over poor quality subcarriers ($\|H(\ell)\|^2 < \tau$) selectively avoiding overhead of retransmission of information symbols corresponding to good quality subcarriers, where τ is threshold on channel norm. The receiver feeds back partial channel state information (PCSI) when each coherent time is elapsed. We assume that due to longer retransmission delay, each retransmission encounters independent channel. First, we present MSCC method under OFDM signaling.

1) *Multiple selective Chase combining*: Motivation of selective Chase combining (SCC) stems from the fact that under OFDM signaling, in the event of CRC failure, receiver can recover from errors by selectively retransmitting information symbols corresponding to the poor quality subcarriers without changing modulation. In SCC, instead of requesting full retransmission for conventional Chase combining, PHY layer requests retransmission of information symbols

corresponding to the poor quality subcarriers and retains observation of the failed data in a buffer. In response to NACK signal, transmitter retransmits requested information symbols. The receiver combines observation of the first transmission and selective retransmission of the poor quality subcarriers for joint decoding in Chase combining fashion and discards buffered observations. This is defined as one round of SCC. Note that one round of conventional CC consists of first transmission followed by full retransmission in the event of CRC failure. The receiver decodes information from observations of first transmission and selective retransmission jointly. Let \mathcal{M} be the maximum allowed number of rounds of CC at MAC layer. At the end of each retransmission round of CC, receiver discards buffered observations and initiates new CC round (if CRC check fails). In selective retransmission, the amount of information to be retransmitted is controlled by threshold τ on the norm of subcarrier gain $\|H(\ell)\|^2$. The threshold τ can be optimized to maximized throughput for given average SNR. In SCC, one round of selective retransmission consists of one round of CC. In [19] and [22], SCC method is presented for SISO-OFDM and MIMO-OFDM under OSTB coded signals, respectively. In this work, we optimize threshold τ on channel quality to maximize throughput for SCC and present throughput analysis. We also propose MSCC method that achieves significant throughput gain over SCC method. We present provide tight upper BER performance bound and throughput analysis of proposed MSCC method.

One retransmission round of MSCC method in PHY consists of at most Ω many conventional CC rounds at MAC layer. In one round of MSCC method, a receiver continues to request retransmission of information symbols over ℓ th subcarrier of a failed packet until sum of norm-square of the subcarrier gains from all iterations of selective retransmission of MSCC method is larger than threshold τ or allowed number of retransmissions is reached. If CRC check is satisfied, selective retransmission and CC rounds are terminated followed by ACK signal to the transmitter. Let J be the count of retransmission rounds of CC at MAC layer. For the I -th iteration of selective retransmission at modulation layer of the J -th retransmission round at MAC layer, if needed at modulation layer, receiver request retransmission of poor quality subcarrier which have $\sum_{i=1}^I \|H_i(\ell)\|^2 < \tau$ for $I = 1, \dots, \Omega$, where Ω is the maximum number of retransmissions considered for joint detection in J -th transmission round at MAC layer. At the end of each conventional CC round at MAC layer, when MSCC method is enabled, receiver jointly detects information bits by combining observations of all previous I selective retransmissions iterations. After $I = \Omega$ iterations of selective retransmission, which constitute one round of proposed MSCC

method, receiver discards buffered observation and initiates new selective retransmission round.

Note that $I = 1$ represents first transmission of a packet and \mathcal{T}_I is set of indices of poor quality subcarriers of the I -th selective retransmission. The set of indices of poor quality subcarriers of $I + 1$ -th selective retransmission \mathcal{T}_{I+1} is subset of set of poor quality indices \mathcal{T}_I of the I -th selective retransmission. That is, $\mathcal{T}_{I+1} \subseteq \mathcal{T}_I$. The receiver stops retransmission of information symbols corresponding to the subcarriers in the iterations of selective retransmission at modulation layer for which $\sum_{i=1}^I \|H_i(\ell)\|^2 \geq \tau$. Let β_I be the cardinality $|\mathcal{T}_I|$ of set \mathcal{T}_I , which represents number of poor subcarriers for the I -th iteration of selective retransmission of the J -th round at MAC layer. The flow graph of MSCC selective retransmissions method is presented in Algorithm 1. We present tight upper bound on uncoded BER and lower bound on throughput

Algorithm 1 MSCC protocol

- 1: $J = 1$ corresponds to the first transmission of the k -th packet
 - 2: Set $I = 1$
 - 3: Detection and CRC check of first iteration of MSCC at PHY layer and J -th round at MAC layer of k -th packet
 - 4: **if** CRC satisfies **then** $k = k + 1$, discard observations and go to 1
 - 5: **if** $I = \Omega$ **then** go to 9
 - 6: Selective retransmission for the β_I subcarriers which have $\sum_{i=1}^I \|H_i(\ell)\|^2 < \tau$ and preservation of observations
 - 7: Joint detection from $N_s + \sum_{i=1}^I \beta_i$ observations
 - 8: Set $I = I + 1$, $J = J + 1$ and go to 4
 - 9: **if** $J = \mathcal{M}$ **then**, discard observations, declare packet loss, $k = k + 1$ and go to 1
 - 10: Discard observations and go to 2
-

of MSCC method under OFDM signaling over Rayleigh fading channel in Section III and Section IV-A, respectively. We also optimized threshold on channel quality that controls amount of information need to be transmitted that maximizes throughput in Section V.

2) *Chase combining with selective retransmission*: Now we discuss proposed CCWS protocol for OFDM signaling. In proposed CCWS method, MAC layer, similar to conventional CC method, initiates full retransmission in the event of packet failure. The proposed CCWS

method is different from convention CC method in the sense that CCWS method employs a selective retransmission corresponding to the poor quality subcarriers at PHY level independent of MAC layer for the first transmission and full retransmission. That is, first transmission of each packet and full retransmission of a failed packet both undergo a selective retransmission of the information symbols corresponding to the poor quality subcarriers at PHY layer. The channel quality ($\|H(\ell)\|^2$) of subcarriers for the first transmission of a packet under CCWS method is compared with threshold τ prior to decoding at PHY layer. The receiver request selective retransmission request to the transmitter and preserves observations of the first transmission of a packet. Then, receiver decodes information by fusing observations of the first transmission and selective retransmission jointly. If CRC check is not satisfied, receiver buffers observation and requests full retransmission of failed packet by sending NACK signal. Similar to the first transmission, receiver initiates selective retransmission of the information symbols corresponding to poor quality subcarriers of full retransmission for which $\|H_c(\ell)\|^2 < \tau$. Note that $H(\ell)$ and $H_c(\ell)$ are the gains of ℓ -th subcarrier for the first transmission and full retransmission, respectively. One round of CCWS consists of a full transmission, a full retransmission and two selective retransmissions. After response of the NACK signal from transmitter, the receiver decodes information by combining observation of the one round of CCWS. If CRC check is satisfied, receiver send ACK signal to the transmitter. Otherwise, receiver clears buffered observations and initiates next round of CCWS. When maximum number of rounds of CCWS are elapsed, packet failure is declared to RLC layer. In order to maximize throughput of CCWS, we optimizes threshold τ on channel quality.

Let β_1 be the number of subcarriers with channel gains $\|H(\ell)\|^2 < \tau$ for the first transmission of CCWS method. The modulation layer initiates selective retransmission of symbols corresponding to the β_1 subcarriers prior to joint detection. Due to selective retransmission and joint detection at modulation layer, BER performance P_{e_1} of the first transmission of CCWS method at MAC layer is same as that of MSCC method with $\Omega = 1$.

Note that channel vector of the poor quality subcarriers of OFDM signaling of CCWS method of the first transmission as a result of selective retransmission is stack of two channel realizations similar to SCC. The decoded $N_s \log_2 M$ bits are processed for CRC check. In the event of CRC failure for the first transmission of a packet, receivers preserves observations of the first transmission ($\beta_1 + N_s$ symbols) and initiates full retransmission request of packet. The modulation

layer initiates selective retransmission of β_2 poor quality subcarrier for which $\|H_c(\ell)\|^2 < \tau$ similar to the first transmission, where $E[\beta_1] = E[\beta_2] = P(\|H_1(\ell)\|^2 < \tau) = P(\|H_c(\ell)\|^2 < \tau) = m$. The joint detection as a result of full retransmission at MAC layer processes $\beta_1 + \beta_2 + 2N_s$ observations to decode $N_s \log_2 M$ bits and achieves BER performance P_{e_2} . Let \mathcal{M} be the maximum number of allowed rounds of transmissions of a packet and J be the round counter for the transmission of the k -th packet of CCWS method. The flow graph of CCWS protocol for the k -th packet is presented in Algorithm 2. We present tight upper bound on uncoded BER and lower bound on throughput of CCWS method under OFDM signaling over Rayleigh fading channel in Section III-B and Section IV-B, respectively. We also optimized threshold on channel quality that controls amount of information need to be transmitted that maximizes throughput in Section V.

Algorithm 2 CCWS protocol

- 1: $J = 1$ corresponds to the first transmission of the k -th packet
 - 2: Selective retransmission of β_1 subcarriers and preservation of N_s observations
 - 3: Joint decoding from $N_s + \beta_1$ observations and CRC check
 - 4: **if** CRC satisfies **then** $k = k + 1$, discard observations and go to 1
 - 5: NACK signal for full retransmission of packet at MAC layer and preservation $\beta_1 + N_s$ observations.
 - 6: Selective retransmission of β_2 poor quality subcarriers corresponding to full retransmission and preservation of N_s observations
 - 7: Joint detection from $\beta_1 + \beta_2 + 2N_s$ observations
 - 8: **if** CRC satisfies **then** $k = k + 1$, discard observations and go to 1
 - 9: **if** $J > \mathcal{M}$ **then** declare packet loss, discard observation, $k = k + 1$ and go to 1
 - 10: $J = J + 1$, discard observation and go to 2
-

Performance analysis of SCC and CCWS is presented next.

III. PERFORMANCE ANALYSIS

In this section, we present tight upper bounds on BER of joint detection for MSCC and CCWS methods at PHY layer. We use these tight upper bound on BER to optimize throughput of the

proposed selective retransmission methods in Section IV. We first derive upper bound on BER of MSCC method discussed in Section II-B1 for $\Omega = 1$ (SCC).

A. BER analysis of SCC

One round of MSCC consists of Ω round of conventional CC method. In case of CRC failure of the first transmission, receiver buffers observations corresponding to the first transmission and requests retransmission of the poor quality subcarrier as shown in Figure 1. Let $h(t)$ and $h_s(t)$ be the channel impulse responses of the first transmission and selective retransmission, respectively shown in Figure 1. We assume that $h(t)$ and $h_s(t)$ are independent due to longer retransmission delay. Note that norm of subcarrier gain and norm of subcarriers channel vector are channel quality measures for SISO-OFDM and SIMO-OFDM systems, respectively. In a typical failed packet, there are fewer error bits and retransmission of potentially culprit bits corresponding to the poor quality sub-carriers can help receiver to recover from error by performing joint detection. In SCC, PHY layer is aware of retransmission due to cross-layer design and performs joint detection for poor quality subcarriers combining observations from the first transmission and subsequent selective retransmission.

Each element of the complex channel vector $H(\ell)$ of the ℓ -th subcarrier follows Gaussian distribution with mean zero and unit variance obtained by applying N_s point DFT matrix on frequency selective channel $h(t)$ of the first transmission of a CC round. The estimate of information symbol over the ℓ -th subcarrier after equalization is

$$\hat{s}(\ell) = s(\ell) + \frac{H^*(\ell)\mathbf{w}(\ell)}{\|H(\ell)\|^2} = s(\ell) + \mathbf{u}(\ell), \quad (2)$$

where $\mathbf{u}(\ell)$ is the effective noise with distribution $\mathcal{N}(0, \|H(\ell)\|^{-2}N_0)$. Note that channel $H(\ell)$ and additive noise $\mathbf{w}(\ell)$ are scalars and vectors for SISO-OFDM and SIMO-OFDM systems, respectively. Selective retransmission method at modulation layer retransmits *partial copy* of symbols by targeting symbols corresponding to the poor quality subcarriers for retransmission. Thus, information symbols corresponding to the subcarriers with channel norm-square $\|H(\ell)\|^2$ less than threshold τ are selected for retransmission. That is, $(\|H(\ell)\|^2 \leq \tau)$. The symbols transmitted over subcarriers for which $\|H(\ell)\|^2 > \tau$ are omitted from retransmission due to the fact that they are less susceptible to impairments. In this work, we search for threshold τ , which

optimizes the amount of information to be retransmitted in order to maximize throughput of an OFDM system.

We denote the outcomes $\|H(\ell)\|^2 > \tau$ and $\|H(\ell)\|^2 \leq \tau$ by the events ξ and ξ^c , respectively. The probabilities of events ξ and ξ^c are

$$P(\xi^c) = P(\|H(\ell)\|^2 \leq \tau) \text{ and } P(\xi) = P(\|H(\ell)\|^2 > \tau),$$

respectively, where random variable $\chi_1 = \|H(\ell)\|^2$ has Chi-square distribution of degree $2n_r$ [24] and $P(\xi^c) = 1 - P(\xi) = P(\chi_1 \leq \tau)$. Now for Rayleigh fading channel,

$$P(\xi^c) = 1 - \exp\left(-\frac{\tau}{2\sigma^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\sigma^2}\right)^k, \quad (3)$$

$$P(\xi) = P(\chi_1 > \tau) = \exp\left(-\frac{\tau}{2\sigma^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\sigma^2}\right)^k, \quad (4)$$

where $\sigma^2 = \frac{1}{2}$ is variance of real and imaginary components of complex channel coefficient of a subcarrier. When event ξ^c occurs, receiver performs joint detection by combining observation of the first transmission and subsequent retransmission. The combined channel response $\mathcal{H}(\ell) = [H^T(\ell) \ H_s^T(\ell)]^T$ is constructed by stacking channel of the first transmission and selective retransmission. If there are β_1 many subcarriers with $\|H(\ell)\|^2 \leq \tau$, then there will be joint detection for β_1 subcarrier for SCC method ($\beta_1 = N_s \implies$ full retransmission). Estimate of information symbol as a result of joint detection is

$$\hat{s}(\ell) = s(\ell) + \underbrace{\|\mathcal{H}(\ell)\|^{-2} \mathcal{H}^H(\ell) \tilde{\mathbf{w}}(\ell)}_{\tilde{\mathbf{u}}(\ell)}, \quad (5)$$

where $\tilde{\mathbf{u}}(\ell) \sim \mathcal{N}_C(0, \|\mathcal{H}(\ell)\|^{-2} N_0)$ and $\tilde{\mathbf{w}}(\ell) \sim \mathcal{N}_C(0, N_0 I_{n_r})$. Note that the random variable $\|\mathcal{H}(\ell)\|^2$ also has Chi-square distribution of degree $4n_r$. Also that $\|\mathcal{H}(\ell)\|^2 = \chi_1 + \chi_2$, where Chi-square random variables χ_1 and $\chi_2 = \|H_s(\ell)\|^2$ are i.i.d. of degree $2n_r$ each. The bit-error probability of joint detection for selective retransmission over Rayleigh fading channel is

$$P_{e_s} = E_H \left[P(\xi^c) P_{e|\xi^c} + P(\xi) P_{e|\xi} \right], \quad (6)$$

where $P_{e|\xi}$ and $P_{e|\xi^c}$ are the conditional bit-error probabilities of single detection and joint detection, respectively. Now we evaluate threshold τ to achieve target BER. Let $P_{e\tau}$ be the bit-error rate and τ be the corresponding channel norm of a subcarrier. Then, for constellation set

of cardinality M , relationship between $P_{e\tau}$ and τ is

$$P_{e\tau} = c \cdot Q \left(\sqrt{g \frac{\tau}{N_0}} \right) \implies \tau = \frac{N_0}{g} \left(Q^{-1} \left(\frac{P_{e\tau}}{c} \right) \right)^2, \quad (7)$$

where c and g are modulation constants of constellation set [23]. Since $Q(\cdot)$ is monotonically decreasing function SNR, in order to achieve BER smaller than $P_{b\tau}$, $\|H(\ell)\|^2 > \tau$. That is, $P_{e|\xi} \leq P_{e\tau}$. Therefore, threshold on the gain of a subcarrier is

$$\|H(\ell)\|^2 \geq \tau = \frac{N_0}{g} \left(Q^{-1} \left(\frac{P_{b\tau}}{c} \right) \right)^2. \quad (8)$$

We use this threshold for selective retransmission at modulation layer to ensure $P_e \leq P_{e\tau}$. In Section V, we search for threshold τ that maximized throughput. The upper bound on probability of error for joint detection of selective Chase combining (SCC) is

$$P_{es} = P(\xi^c) \cdot c \cdot E_{H|\xi^c} \left[Q \left(\sqrt{g \frac{\chi_1}{N_0}} \right) \right] + P(\xi) \cdot c \cdot E_{H|\xi} \left[Q \left(\sqrt{g \frac{\chi_1 + \chi_2}{N_0}} \right) \right], \quad (9)$$

where $E_{H|\xi^c}$ and $E_{H|\xi}$ are conditional expectations. The subscript s in (9) represent selective Chase combining method. The conditional probability density function $f_{\chi_1|\xi^c}(x_1)$ of $f_{\chi_1}(x_1)$ when $\chi_1 > \tau$ is $f_{\chi_1|\xi^c}(x_1) = \frac{f_{\chi_1}(x_1)}{P(\xi^c)}$, where $\chi_1 > \tau$. In order to solve first term of (9) [25], we have

$$E_{H|\xi^c} \left[Q \left(\sqrt{g \frac{\chi_1}{N_0}} \right) \right] = \int_{\tau}^{\infty} Q \left(\sqrt{g \frac{x_1}{N_0}} \right) \frac{f_{\chi_1}(x_1)}{P(\xi^c)} dx_1. \quad (10)$$

Similarly,

$$E_{H|\xi} \left[Q \left(\sqrt{g \frac{\chi_1 + \chi_2}{N_0}} \right) \right] = \int_{x_1=0}^{\tau} \int_{x_2=0}^{\infty} Q \left(\sqrt{g \frac{x_1 + x_2}{N_0}} \right) \frac{f_{\chi_1}(x_1) f_{\chi_2}(x_2)}{P(\xi)} dx_2 dx_1. \quad (11)$$

The upper bound on P_{es} in (9) using approximation of Q-function [26], (10) and (11) can be written as [22], [25]

$$\begin{aligned} P_{es} &\leq \frac{c}{12} \int_{\tau}^{\infty} \exp(-g \frac{x_1}{2N_0}) f_{\chi_1}(x_1) dx_1 + \frac{c}{4} \int_{\tau}^{\infty} \exp(-g \frac{4x_1}{3.2N_0}) f_{\chi_1}(x_1) dx_1 \\ &\quad + \frac{c}{12} \int_0^{\tau} \exp(-g \frac{x_1}{2N_0}) f_{\chi_1}(x_1) dx_1 \cdot \int_0^{\infty} \exp(-g \frac{x_2}{2N_0}) f_{\chi_2}(x_2) dx_2 \\ &\quad + \frac{c}{4} \int_0^{\tau} \exp(-g \frac{4x_1}{3.2N_0}) f_{\chi_1}(x_1) dx_1 \cdot \int_0^{\infty} \exp(-g \frac{4x_2}{3.2N_0}) f_{\chi_2}(x_2) dx_2 \end{aligned} \quad (12)$$

Simplifying (12), we have upper bound on probability of error of the joint detection as follows

$$\begin{aligned}
P_{e_s} \leq & \frac{c}{12} \left(\frac{\rho}{\sigma} \right)^{2n_r} \exp \left(-\frac{\tau}{2\rho^2} \right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2} \right)^k + \frac{c}{4} \left(\frac{\rho_1}{\sigma} \right)^{2n_r} \exp \left(-\frac{\tau}{2\rho_1^2} \right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2} \right)^k \\
& + \frac{c}{12} \left(\frac{\rho}{\sigma} \right)^{4n_r} \left(1 - \exp \left(-\frac{\tau}{2\rho^2} \right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2} \right)^k \right) \\
& + \frac{c}{4} \left(\frac{\rho_1}{\sigma} \right)^{4n_r} \left(1 - \exp \left(-\frac{\tau}{2\rho_1^2} \right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2} \right)^k \right),
\end{aligned} \tag{13}$$

where $\rho = \sqrt{\frac{\sigma^2 N_o}{g\sigma^2 + N_o}}$, $\rho_1 = \sqrt{\frac{\sigma^2 N_o}{g_1\sigma^2 + N_o}}$ and $g_1 = \frac{4g}{3}$. For 4-QAM constellation $g = 2$ and $c = \frac{2}{\log_2 M}$ [24]. Note that in (13), P_{e_j} is function of τ , which controls amount of information to be retransmitted in selective retransmission. The probability of error of the first transmission with n_r receiver antenna can be derived from (13) by setting parameter $\tau \rightarrow 0$, which means that probability of selecting a subcarrier for retransmission is almost zero. For $\tau \rightarrow 0$, (13) becomes

$$P_e \leq \frac{c}{12} \left(\frac{\rho}{\sigma} \right)^{2n_r} + \frac{c}{4} \left(\frac{\rho_1}{\sigma} \right)^{2n_r}, \tag{14}$$

which is probability of error of the first transmission with n_r receive antennas. Similarly, for full retransmission, $\tau \rightarrow \infty$ and BER upper bound can be achieved by setting $\tau = \infty$ in (13). Thus,

$$P_{e_f} \leq \frac{c}{12} \left(\frac{\rho}{\sigma} \right)^{4n_r} + \frac{c}{4} \left(\frac{\rho_1}{\sigma} \right)^{4n_r}. \tag{15}$$

In low SNR regime, receiver needs more information during retransmission to recover from errors. Therefore, threshold τ on channel norm has large value in low SNR regime. Throughput η is function of probability of error of first transmission P_e and probability of error of joint detection P_{e_s} , which is function of SNR and threshold τ . We use (13) and (14) to search of optimal value of τ to maximize throughput of SCC method in Section IV. Now we provide BER analysis of CCWS method.

B. BER analysis of CCWS

The probability of bit-error P_{e_1} of the first transmission of CCWS is in fact P_{e_s} for SCC given in (13). There are two outcomes from the first transmission related to the ℓ -th OFDM subcarrier denoted by ξ and ξ^c with probabilities $P(\xi) = P(\|H(\ell)\|^2 > \tau)$ and $P(\xi^c) = P(\|H(\ell)\|^2 \leq \tau)$.

$\tau) = 1 - P(\xi)$ similar to SCC. It is important to note that BER P_{e_1} of the first transmission of CCWS is upper bounded by (13). That is, $P_{e_1} = P_{e_s}$.

Now we evaluate upper bound on BER for joint detection when CRC failure occurs for CCWS method. Let $H_c(\ell)$ be the channel of the ℓ -th subcarrier during full retransmission of packet of Chase combining protocol. Similar to SCC method, modulation layer initiates retransmission of the poor subcarriers $\|H_c(\ell)\|^2 < \tau$ for the full retransmission. We denote the event $\|H_c(\ell)\|^2 \leq \tau$ by ξ_c . The subscript c represents retransmission of Chase combining and superscript with event ξ_c represent complement of ξ_c . The channel realizations during first transmission and conventional Chase combining are $H(\ell)$ and $H_c(\ell)$, respectively, are i.i.d. and $P(\xi) = P(\|H(\ell)\|^2 > \tau) = P(\xi_c) = P(\|H_c(\ell)\|^2 > \tau)$. In response to full retransmission of failed packet, receiver combines $\beta_s + N_s$ observations of the first transmission and $\beta_c + N_s$ of full retransmission for joint detection. Note that β_c is the number of poor subcarriers retransmitted at modulation layer due to full retransmission at data link layer. Probability of error of joint detection is as follows:

$$P_{e_2} = E_H [P(\xi_1)P_{e|\xi_1} + P(\xi_2)P_{e|\xi_2} + P(\xi_3)P_{e|\xi_3} + P(\xi_4)P_{e|\xi_4}], \quad (16)$$

where events ξ_1, ξ_2, ξ_3 and ξ_4 defined as follows:

- 1) Event ξ_1 occurs when $\|H(\ell)\|^2 > \tau$ and $\|H_c(\ell)\|^2 > \tau$. The resulting joint channel for joint detection of CCWS into $\mathcal{H}_1(\ell) = [H^T(\ell) \ H_c^T(\ell)]^T$, where $H(\ell)$ and $H_c(\ell)$ are i.i.d. channel realization with Gaussian distribution of zero mean and unit variance. The channel norm $\|\mathcal{H}_1(\ell)\|^2 = \|H(\ell)\|^2 + \|H_c(\ell)\|^2$ has Chi-square distribution.
- 2) Event ξ_2 occurs when $\|H(\ell)\|^2 \leq \tau$ and $\|H_c(\ell)\|^2 > \tau$. The resulting joint channel response for joint detection of CCWS into $\mathcal{H}_2(\ell) = [H^T(\ell) \ H_s^T(\ell) \ H_c^T(\ell)]^T$.
- 3) Event ξ_3 occurs when $\|H(\ell)\|^2 > \tau$ and $\|H_c(\ell)\|^2 \leq \tau$. The resulting joint channel response for joint detection of CCWS into $\mathcal{H}_3(\ell) = [H^T(\ell) \ H_c^T(\ell) \ H_{sc}^T(\ell)]^T$, where $H_{sc}(\ell)$ is channel gain of the ℓ -th subcarrier selected for retransmission during full retransmission of failed packet at data link layer.
- 4) Event ξ_4 occurs when $\|H(\ell)\|^2 \leq \tau$ and $\|H_c(\ell)\|^2 \leq \tau$. The resulting joint channel for joint detection of CCWS into $\mathcal{H}_4(\ell) = [H^T(\ell) \ H_s^T(\ell) \ H_c^T(\ell) \ H_{sc}^T(\ell)]^T$, where $H_s(\ell)$ and $H_{sc}(\ell)$ are the channels corresponding to the selective channels from the first transmission and full retransmission of CCWS method. Note that random variables $\|H_s(\ell)\|^2$ and $\|H_{sc}(\ell)\|^2$ are i.i.ds. with Chi-square distribution of degree $2n_r$ each.

Second and third terms in (16) are equivalent due to the fact that $P(\xi_2) = P(\xi_3)$ and random variables $\|\mathcal{H}_2(\ell)\|$ and $\|\mathcal{H}_3(\ell)\|$ are i.i.ds. Therefore, $E_H[P(\xi_2)P_{e|\xi_2} + P(\xi_3)P_{e|\xi_3}] = 2E_H[P(\xi_2)P_{e|\xi_2}]$. Note that all channel realizations of the ℓ -th subcarrier of an OFDM system are i.i.d. with Gaussian distribution of mean zero and unit variance. In order to achieve joint upper bound on BER for joint detection of CCWS method, we rewrite (16) as follows:

$$P_{e2} = cE_H \left[P(\xi_1)Q \left(\sqrt{\frac{g\|\mathcal{H}_1(\ell)\|^2}{N_0}} \right) + 2P(\xi_2) \cdot Q \left(\sqrt{\frac{g\|\mathcal{H}_2(\ell)\|^2}{N_0}} \right) + P(\xi_4)Q \left(\sqrt{\frac{g\|\mathcal{H}_4(\ell)\|^2}{N_0}} \right) \right]. \quad (17)$$

Note that $\|\mathcal{H}_1(\ell)\|^2 = \chi + \chi_c$ in the first term of (17) is sum of two i.i.d. chi-square random variables, where $\chi > \tau$ and $\chi_c > \tau$. Using approximation of Q-function in [26] and, following (10) and (11), we have

$$\begin{aligned} E_H [P(\xi_1)P_e|\xi_1] &= cE_H \left[P(\xi_1)Q \left(\sqrt{\frac{g\|\mathcal{H}_1(\ell)\|^2}{N_0}} \right) \right] \\ &\leq \frac{c}{12} \int_{\tau}^{\infty} \exp(-g\frac{x}{2N_0})f_X(x)dx \cdot \int_{\tau}^{\infty} \exp(-g\frac{x_c}{2N_0})f_{X_c}(x_c)dx_c + \\ &\quad \frac{c}{4} \int_{\tau}^{\infty} \exp(-g\frac{4x}{3.2N_0})f_X(x)dx \int_{\tau}^{\infty} \exp(-g\frac{4x_c}{3.2N_0})f_{X_c}(x_c)dx_c \quad (18) \\ &= \frac{c}{12} \left(\frac{\rho}{\sigma} \right)^{4n_r} \left(\exp \left(-\frac{\tau}{2\rho^2} \right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2} \right)^k \right)^2 + \\ &\quad \frac{c}{4} \left(\frac{\rho_1}{\sigma} \right)^{4n_r} \left(\exp \left(-\frac{\tau}{2\rho_1^2} \right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2} \right)^k \right)^2. \end{aligned}$$

Similarly,

$$\begin{aligned} 2E_H [P(\xi_2)P_e|\xi_2] &= 2cE_H \left[P(\xi_2)Q \left(\sqrt{\frac{g\|\mathcal{H}_2(\ell)\|^2}{N_0}} \right) \right] \\ &\leq \frac{c}{6} \int_0^{\tau} \exp(-g\frac{x}{2N_0})f_X(x)dx \cdot \int_0^{\infty} \exp(-g\frac{x_s}{2N_0})f_{X_s}(x_s)dx_s \int_{\tau}^{\infty} \exp(-g\frac{x_c}{2N_0})f_{X_c}(x_c)dx_c + \\ &\quad \frac{c}{2} \int_0^{\tau} \exp(-g\frac{4x}{3.2N_0})f_X(x)dx \cdot \int_0^{\infty} \exp(-g\frac{4x_s}{3.2N_0})f_{X_s}(x_s)dx_s \int_{\tau}^{\infty} \exp(-g\frac{4x_c}{3.2N_0})f_{X_c}(x_c)dx_c, \quad (19) \end{aligned}$$

where $\chi < \tau$, $\chi_s \in \mathcal{R}$ and $\chi_c > \tau$. Simplifying (19), we have

$$2E_H [P(\xi_2)P_e|\xi_2] \leq \frac{c}{6} \left(\frac{\rho}{\sigma}\right)^{6n_r} \left(1 - \exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right) \left(\exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right) +$$

$$\frac{c}{2} \left(\frac{\rho_1}{\sigma}\right)^{6n_r} \left(1 - \exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right) \left(\exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right). \quad (20)$$

Also, it can be shown that

$$E_H [P(\xi_4)P_e|\xi_4] = cE_H \left[P(\xi_4)Q \left(\sqrt{\frac{g\|\mathcal{H}_4(\ell)\|^2}{N_0}} \right) \right]$$

$$\leq \frac{c}{12} \left(\frac{\rho}{\sigma_h}\right)^{8n_r} \left(1 - \exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right)^2 +$$

$$\frac{c}{4} \left(\frac{\rho_1}{\sigma_h}\right)^{8n_r} \left(1 - \exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right)^2, \quad (21)$$

where $\chi < \tau$, $\chi_s \in \mathcal{R}$, $\chi_c < \tau$ and $\chi_{sc} \in \mathcal{R}$. Now using (18), (20) and (21) in (17), we have

$$P_{e2} \leq \frac{c}{12} \left(\frac{\rho}{\sigma}\right)^{4n_r} \left(\exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right)^2 + \frac{c}{4} \left(\frac{\rho_1}{\sigma}\right)^{4n_r} \left(\exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right)^2 +$$

$$\frac{c}{6} \left(\frac{\rho}{\sigma}\right)^{6n_r} \left(\exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right) \left(1 - \exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right) +$$

$$\frac{c}{2} \left(\frac{\rho_1}{\sigma}\right)^{6n_r} \left(\exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right) \left(1 - \exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right) +$$

$$\frac{c}{12} \left(\frac{\rho}{\sigma_h}\right)^{8n_r} \left(1 - \exp\left(-\frac{\tau}{2\rho^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho^2}\right)^k\right)^2$$

$$+ \frac{c}{4} \left(\frac{\rho_1}{\sigma_h}\right)^{8n_r} \left(1 - \exp\left(-\frac{\tau}{2\rho_1^2}\right) \sum_{k=0}^{n_r-1} \frac{1}{k!} \left(\frac{\tau}{2\rho_1^2}\right)^k\right)^2. \quad (22)$$

In next section, we present throughput analysis and optimization with respect parameter τ for SCC and CCWS.

IV. THROUGHPUT ANALYSIS

Now we present throughput analysis of the proposed SCC and CCWS methods. In throughput analysis, we consider infinite many rounds of retransmission. One round of retransmission

consists of first transmission followed by a retransmission. The receiver preserves observation corresponding to the first transmission for joint detection. We consider joint detection for retransmission by combining observation from the first transmission and subsequent *partial or full* retransmission. In practice, transceiver pair continues retransmission rounds until error free packet is received or maximum number of retransmission rounds are reached. For throughput analysis, we follow conventional definition of throughput η , which is ratio of error-free information bits received k to the total number of bits transmitted n ($\eta = \frac{k}{n}$). Let P_e and P_{e_s} be the bit-error probabilities of the first transmission and joint detection followed by retransmission, respectively. Assuming that each bit in the frame is independent, probability of receiving an error-free packet of length L_f with probability of bit-error P_e is $p_c = (1 - P_e)^{L_f}$. Then probability of receiving a bad packet is $p_e = 1 - p_c$. Similarly, for joint detection, $p_{c_s} = (1 - P_{e_s})^{L_f}$ and $p_{e_s} = 1 - p_{c_s}$. Next, we present throughput analysis of SCC method.

A. Throughput analysis of SCC

Now we provide throughput analysis of SCC method. In response to the NACK signal in the event of packet failure, receiver initiates selective retransmission of poor quality subcarriers instead of full retransmission. Let m be the fraction of subcarriers selected for retransmission. That is, at a given SNR, fraction $m = P(\|H(\ell)\|^2 \leq \tau)$. In SCC, each retransmission round consists of first transmission followed by *partial* retransmission resulting in to transmission of $k(1 + m)$ bits in each round. Let p_c and p_{c_s} be the probabilities of receiving correct frame from the first transmission and joint detection of SCC method, respectively. The probabilities of receiving bad frame from the first transmission and joint detection of SCC are p_e and p_{e_s} , respectively, for SCC. The probabilities of frame error p_e and p_{e_s} are functions of BER given in (13) and (14), respectively. Thus, expected number of information bits n to be transmitted to receive k error-free bits are [27]

$$\begin{aligned}
 n = & kp_c + k(1 + m)p_e p_{c_s} + k(2 + m)p_e p_{e_s} p_c + k(2 + 2m)p_e^2 \\
 & p_{e_s} p_{c_s} + k(3 + 2m)p_e^2 p_{e_s}^2 p_c + k(3 + 3m)p_e^3 p_{e_s}^2 p_{c_s} + k(4 + 3m) \\
 & p_e^3 p_{e_s}^3 p_c + k(4 + 4m)p_e^4 p_{e_s}^3 p_{c_s} + k(5 + 4m)p_e^4 p_{e_s}^4 p_c + \dots
 \end{aligned} \tag{23}$$

Now rearranging (23), we have

$$\begin{aligned}
n &= kp_c \left(1 + (2+m)p_\epsilon p_{\epsilon_s} + (3+2m)p_\epsilon^2 p_{\epsilon_s}^2 + (4+3m)p_\epsilon^3 p_{\epsilon_s}^3 + (5+4m)p_\epsilon^4 p_{\epsilon_s}^4 + \dots \right) \\
&\quad + kp_\epsilon p_{c_s} \left((1+m) + (2+2m)p_\epsilon p_{\epsilon_s} + (3+3m)p_\epsilon^2 p_{\epsilon_s}^2 + (4+4m)p_\epsilon^3 p_{\epsilon_s}^3 + \dots \right) \\
&= kp_c \left(1 + (2+m)\alpha + (3+2m)\alpha^2 + (4+3m)\alpha^3 + \dots \right) \\
&\quad + kp_\epsilon p_{c_s} \left((1+m) + (2+2m)\alpha + (3+3m)\alpha^2 + (4+4m)\alpha^3 + (5+5m)\alpha^4 + \dots \right) \\
&= k.p_c(1+m\alpha)(1+2\alpha+3\alpha^2+4\alpha^3+\dots) \\
&\quad + kp_\epsilon p_{c_s}(1+m) \left(1+2\alpha+3\alpha^2+4\alpha^3+\dots \right)
\end{aligned} \tag{24}$$

where $\alpha = p_\epsilon p_{\epsilon_s}$. Note that $1 + 2\alpha + 3\alpha^2 + 4\alpha^3 + \dots = \frac{1}{(1-\alpha)^2}$ [28]. Therefore,

$$n = \frac{kp_c(1+m\alpha) + p_\epsilon p_{c_s}(1+m)}{(1-\alpha)^2}. \tag{25}$$

Thus, throughput of SCC methods is

$$\eta_s = \frac{k}{n} = \frac{(1-\alpha)^2}{p_c(1+m\alpha) + p_\epsilon p_{c_s}(1+m)}. \tag{26}$$

It is clear from (13), (14) and (26) that throughput $\eta_s = f(\tau, SNR)$. Next, we present throughput analysis of CCWS method.

B. Throughput analysis of CCWS

In CCWS method, modulation layer executes selective retransmission for first transmission of a packet and full retransmission in the event of CRC failure. Thus, for each of the first transmission and subsequent full retransmission of OFDM symbol initiated by Chase combining protocol at data-link layer, $k(1+m)$ information bits are transmitted, where additional km bits are the results of selective retransmission for the first transmission and full retransmission. Similar to SCC method, P_{e_1} and P_{e_2} be the probability of error for the first transmission and joint detection corresponding to the retransmission. Since CCWS method employs selective Chase combining during first transmission, the bit-error probability of the first transmission is given in (13). The bit-error rate P_{e_2} for joint detection CCWS is given in (22), respectively. We denote probabilities of receiving error-free frame of length L_f for the first transmission and selective

retransmission by p_{c_1} and p_{c_2} , respectively. The corresponding probabilities of receiving incorrect frame for first transmission and joint detection in the event of packet failure are $p_{\epsilon_1} = 1 - p_{c_1}$ and $p_{\epsilon_2} = 1 - p_{c_2}$, respectively. The expected number of information bits transmitted to deliver error-free k information bits for CCWS method are [27]

$$\begin{aligned}
n &= k \cdot (1 + m)p_{c_1} + 2k(1 + m)p_{\epsilon_1} \cdot p_{c_2} + k3(1 + m)p_{\epsilon_1}p_{\epsilon_2}p_{c_1} + \\
&\quad 4k(1 + m)p_{\epsilon_1}^2p_{\epsilon_2}p_{c_2} + k \cdot 5(1 + m)p_{\epsilon_1}^2p_{\epsilon_2}^2p_{c_1} + 6k(1 + m) \\
&\quad p_{\epsilon_1}^3p_{\epsilon_2}^2p_{c_2} + k \cdot 7(1 + m)p_{\epsilon_1}^3p_{\epsilon_2}^3p_{c_1} + \dots \\
&= k \cdot (1 + m)p_{c_1} \left(1 + 3p_{\epsilon_1}p_{\epsilon_2} + 5p_{\epsilon_1}^2p_{\epsilon_2}^2 + \dots \right) + \\
&\quad 2k(1 + m)p_{\epsilon_1}p_{c_2} \left(1 + 2p_{\epsilon_1}p_{\epsilon_2} + 3p_{\epsilon_1}^2p_{\epsilon_2}^2 + \dots \right).
\end{aligned} \tag{27}$$

Let $\alpha = p_{\epsilon_1}p_{\epsilon_2}$, then

$$\begin{aligned}
n &= k(1 + m)p_{c_1} \left(1 + 3\alpha + 5\alpha^2 + 7\alpha^3 + \dots \right) + \\
&\quad 2k(1 + m)p_{\epsilon_1} \cdot p_{c_2} \left(1 + 2\alpha + 3\alpha^2 + 4\alpha^3 + \dots \right).
\end{aligned} \tag{28}$$

After simplification of (28), we have [28]

$$n = k(1 + m)p_{c_1} \frac{1 + \alpha}{(1 - \alpha)^2} + 2k(1 + m)p_{\epsilon_1}p_{c_2} \frac{1}{(1 - \alpha)^2}. \tag{29}$$

Throughput of CCWS method is

$$\eta_c = \frac{k}{n} = \frac{(1 - \alpha)^2}{\left(p_{c_1}(1 + \alpha) + 2(1 - p_{c_1})p_{c_2} \right) (1 + m)}. \tag{30}$$

Note that similar to the throughput η_s for SCC method, throughput η_c of CCWS is also function of parameter τ that controls the information to be transmitted during selective retransmission. The parameter τ can be optimize to maximize throughput under OFDM signaling. Next, we discuss search for optimal τ for SCC and CCWS to enhance throughput of OFDM transceiver.

V. THROUGHPUT OPTIMIZATION

In this section, we optimize throughput of the proposed selective retransmission methods at modulation layer. The amount of information that a receiver request to the transmitter in the event of a packet failure has direct impact on the throughput of the transceiver. Most of the time, especially in high SNR regime, receiver can recover from bit errors by receiving little more information and employing joint detection. In selective retransmission at modulation level,

threshold τ on channel norm of a subcarrier is measure of quality of a channel. By choosing proper threshold τ , receiver can request minimum information needed to recover from errors for a failed packet. The threshold τ is function of SNR and modulations such as 4-QAM and 16-QAM. It is clear from (26) and (30) that throughput of SCC and CCWS methods, respectively, are function of frame-error rate (FER). Furthermore, FER is not a linear or quadratic function of SNR and parameter τ . Now we write unconstrained optimization problem for throughput η with respect to parameter τ as follows:

$$\tau_0 = \arg \max_{\tau} \eta = f(\tau, SNR). \quad (31)$$

Since throughput η is non-convex function in parameter τ , optimal τ that maximizes throughput η for each SNR can be computed off-line using exhaustive search. Thus, a table of optimal threshold τ to maximize throughput for SNR operating points can be generated using throughput expression in (26) and (30) for SCC and CCWS methods, respectively. Note that each retransmission method has different throughput function $\eta = f(\tau, SNR)$. For example, analytical throughput function for SCC and CCWS is given in (26) and (30), respectively. In Section VI, we maximize $\eta = f(\tau, SNR)$ with respect to parameter τ . Note that parameter τ appears in probability of frame error p_e , which is function of probability of bit-error presented in Section (III). Although $\eta = f(\tau, SNR)$ is not linear or quadratic function of τ at a given SNR, Figure 2 shows that for a given SNR, τ_o that maximizes throughput η is unique for SCC method. The optimal τ_o can be computed off-line from throughput lower bound for SCC and CCWS using (26) and (30), respectively. Based on channel condition, amount of information to be transmitted can be controlled using vector τ_o . Figure 3 shows optimal threshold τ_o for SNR points that maximizes throughput CCWS method. In low SNR regime, throughput η is more sensitive to threshold τ as compared to high SNR regime. This is due to the fact that in high SNR regime, very few errors occur during first transmission resulting in to fewer retransmissions. We use threshold vector τ_o in next section to compute throughput.

VI. SIMULATION

Now we present efficiency for aforementioned SCC, CCWS and MSCC methods for optimized threshold τ_o on the throughput of OFDM systems. In simulation setup, we consider 4-QAM constellation for OFDM signaling under Rayleigh fading frequency selective channel. We use

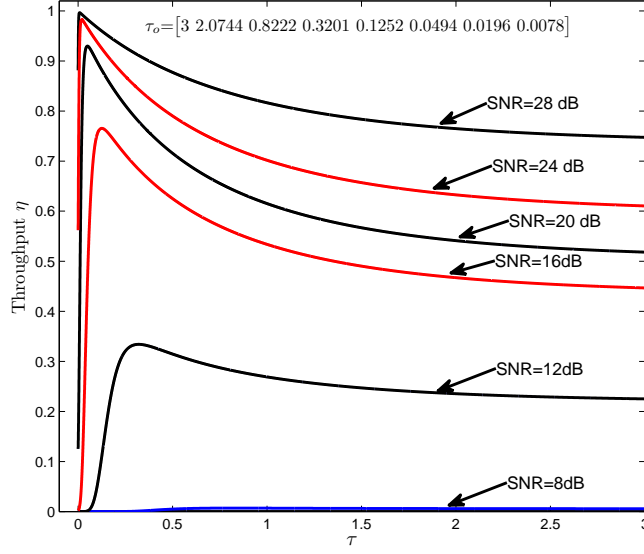


Fig. 2. Optimal τ_o for different SNR operating point that maximizes analytical throughput of SCC method in (26).

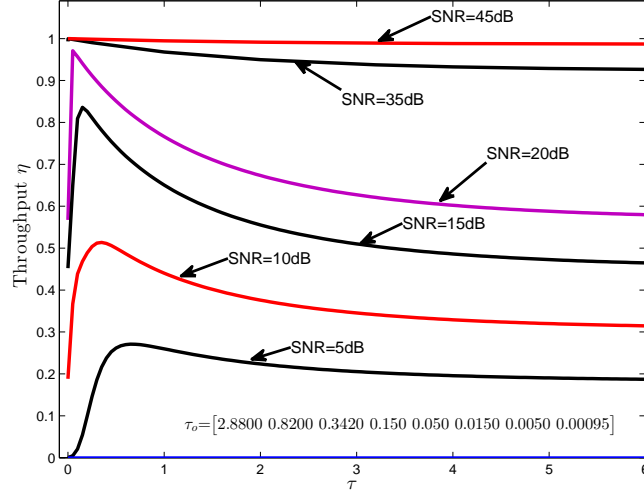


Fig. 3. Optimal τ_o for different SNR operating point that maximizes analytical throughput of CCWS method in (30).

OFDM signaling with $N_s = 512$ subcarriers for 10-tap frequency selective channel. Each complex OFDM channel realization has Gaussian distribution with zero-mean and unit variance ($\sigma_h^2 = 1$). We assume block fading channel in quasi-static fashion such that channel remains highly

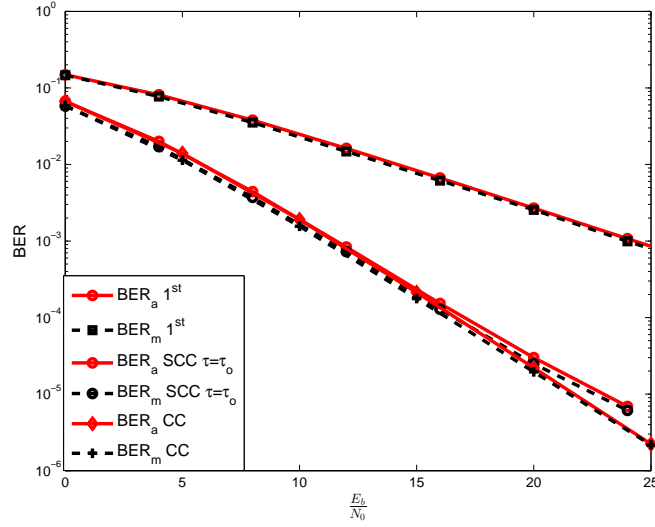


Fig. 4. BER performance comparison of analytical bit-error rate in (13) and Monte Carlo simulation for SCC method of SISO-OFDM system. In analytical and simulation results, parameter $\tau = 0$, τ_f and τ_o are used to target BER without selective retransmission(single), full retransmission and optimize throughput, respectively, for SCC method.

correlated during transmission of one OFDM symbol. First, we present comparison of analytical BER upper bound and BER from Monte Carlo simulation denoted by BER_a and BER_m , respectively. We also provide throughput results of SCC and CCWS methods in comparison with conventional Chase combining method. We denote analytical and simulation throughput by η_a and η_m , respectively. In order to maximize throughput, threshold τ on channel norm of OFDM subcarriers is optimized for each SNR point of SCC and CCWS protocols. We analytically compute threshold vector τ_o off-line to maximize throughput of SCC and CCWS methods from the analytical throughput using (26) and (30), respectively. We also demonstrate that throughput gain our proposed CCWS and MSCC methods hold under CC-HARQ method at MAC layer. We consider half-rate LDPC code (648, 324) to evaluate efficacy of CCWS and MSCC methods. We denote CCWS and MSCC methods with FEC enabled by CCWS-HARQ and MSCC-HARQ in simulation results.

First, we compare BER performance for single (first) transmission and subsequent retransmission in the event of failed packet. In particular, we compare BER of full retransmission (Chase combining) and SCC in Figure 4. For analytical and Monte Carlo runs of SCC, we use optimal threshold vector τ_o that maximizes throughput. In Figure 4, we present BER comparison of upper

bound and BER from Monte Carlo simulation of SCC under SISO-OFDM system using optimal threshold vector τ_o . Note that BER performance of SCC with threshold τ_o and τ_f is similar, where threshold τ_f targets BER of SCC equals full retransmission. Figure 5 compares threshold vector τ_o that optimizes throughput of SCC and CCWS at different SNR points and threshold vector τ_f to target BER of SCC and CCWS equals the corresponding full retransmission. Difference between vectors τ_o and τ_f is marginal at moderate and high SNR. At high SNR, BER of SCC with threshold τ_o is marginally higher than that of BER for threshold τ_f (BER of full retransmission). It is clear from Figure 4 that retransmission of partial information corresponding to poor quality subcarriers achieves BER of full retransmission. The threshold τ on channel norm of subcarriers controls amount of partial information to be retransmitted. Figure 6 compares BER performance

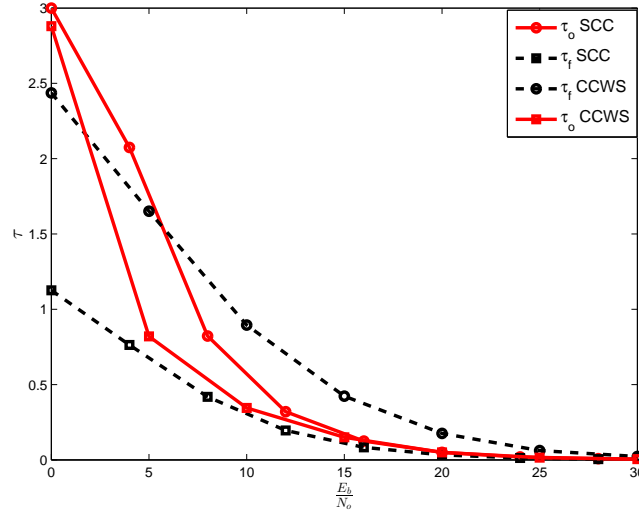


Fig. 5. Threshold comparison τ_f and τ_o corresponding to target BER equals full retransmission and optimal throughput for both SCC and CCWS protocols, respectively.

of CCWS method with conventional CC method for different values of threshold vector τ on channel norm of subcarriers. First transmission of packet under CCWS method, which employs selective retransmission at modulation level, achieves BER of full retransmission with threshold vector τ_o as shown in Figure 6. Threshold vector τ_o for CCWS method optimizes throughput of CCWS. The comparison of τ_o for SCC and CCWS is presented in Figure 5. Figure 6 also reveals that using threshold τ_o that maximizes throughput of CCWS method, BER of joint detection under CCWS method is similar to the BER of four receive antennas. Note that threshold τ_∞ implies

retransmissions of all subcarriers and τ_∞ for CCWS in fact is equivalent to four transmissions of a packets or four receive antennas. In Figure 6, we also compare analytical BER (BER_a) of joint detection given in (22) with Monte Carlo BER (BER_m) of CCWS for threshold vector τ_o . It is clear from Figure 4 and Figure 6 that (13) and (22) are tight BER upper bounds for joint detections of SCC and CCWS methods, respectively. In CCWS method, threshold τ_o is computed using (31) to maximize throughput η . Similar to SCC method, we compute threshold τ_f that targets BER of CCWS equals to the BER of joint detection of four transmissions. For CCWS method, BER_m for threshold vector τ_f is lower than BER_m for threshold vector τ_o .

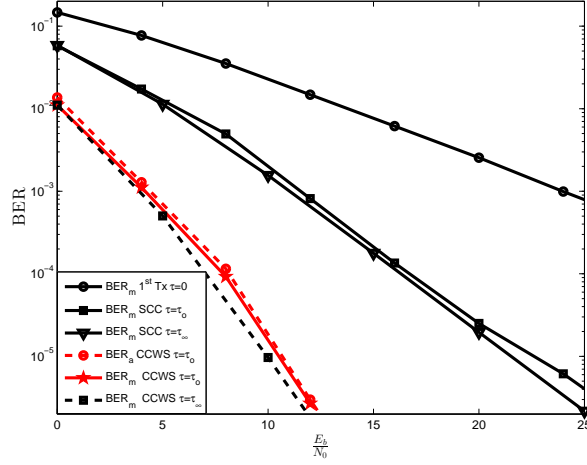


Fig. 6. BER performance comparison of analytical bit-error rate in (22) and Monte Carlo simulation for CCWS method of SISO-OFDM system. In analytical and simulation results, parameter $\tau = 0, \tau_f$ and τ_o are used to target BER corresponding to full retransmission and optimize throughput for both first transmission and retransmission, respectively, for CCWS method.

Throughput is the key performance metric of a communication system and throughput optimization of selective retransmission based on channel quality for OFDM system is the main focus of this work. For throughput computation, we use standard definition of throughput of a communication system [2] as

$$\eta = \frac{\text{Error-free bits received}}{\text{Total bits transmitted}} = \frac{k}{n}. \quad (32)$$

Now we provide throughput results to demonstrate the efficacy of selective retransmission on spectral efficiency for SCC and CCWS methods at modulation layer. We use throughput gain of optimized τ_o for SCC and CCWS methods over conventional Chase combining method as a

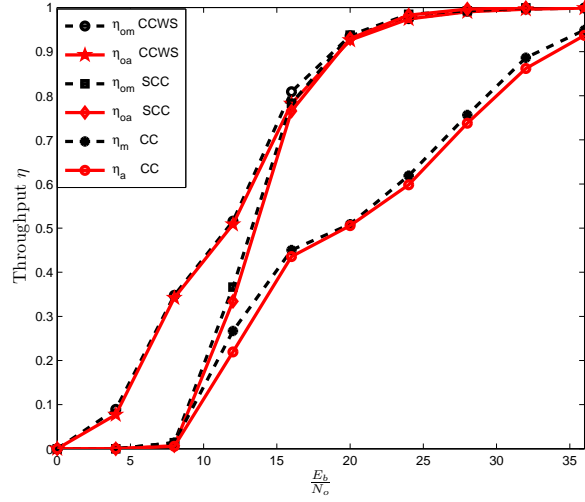


Fig. 7. Simulation and Analytical Throughput comparison for SCC, CCWS and CC using (26) and (30) for τ_o .

measure of spectral efficiency. Figure 7 provides throughput comparison of conventional Chase combining, SCC and CCWS methods. Monte Carlo throughput (simulation) η_{om} for SCC and CCWS is computed using their respective optimal thresholds τ_o . The throughput performance of both SCC and CCWS is much higher than conventional Chase combining method. Throughput in Figure 7 also reveals that CCWS outperforms SCC in low SNR regime. This is due to the fact that at low SNR, joint detection using more retransmission improves throughput of the communication systems. Figure 7 also compares analytical throughput η_{oa} and simulation throughput η_{om} of SCC and CCWS methods. Analytical throughput η_{oa} of SCC and CCWS is computed using (26) and (30), respectively. The throughput expressions for SCC and CCWS in (26) and (30), respectively, provide tight lower bounds on throughput as shown in Figure 7.

Now we present impact of thresholds τ_o and τ_f on the throughput of SCC and CCWS methods. Figure 8 compares simulation results of throughput η_{om} and η_{fm} corresponding to the optimal threshold τ_o that maximizes throughput and threshold τ_f that achieves BER of full retransmission of the proposed methods (SCC and CCWS). The simulation results suggest that throughput of SCC method for η_{om} and η_{fm} are similar. Throughput for η_{om} are marginally better than for η_{fm} . However, throughput η_{om} of CCWS with optimal threshold vector τ_o is higher than throughput η_{fm} of CCWS with threshold vector τ_f that achieves BER of full retransmission.

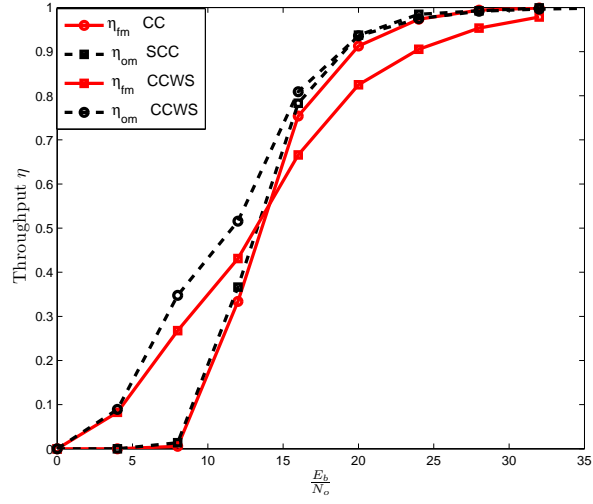


Fig. 8. Throughput comparison of Optimal SCC and CCWS using Target BER of Full retransmission $\tau = 0$ Vs optimal SCC form Figure 5.

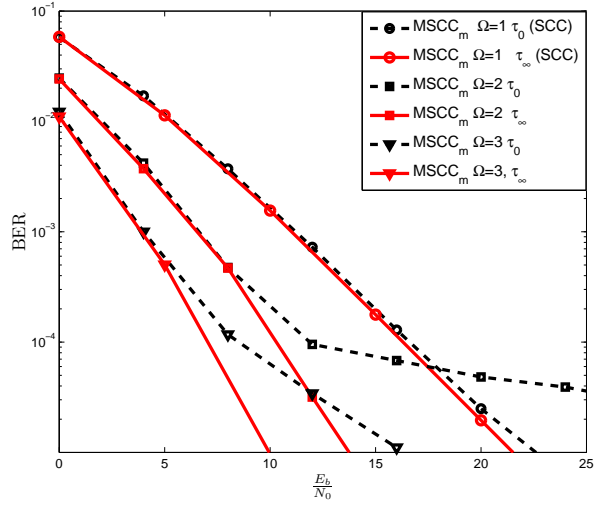


Fig. 9. BER comparison of SCC and MSCC for τ_0 and τ_∞ .

BER of MSCC for retransmission count $\Omega = 1, 2$ and 3 for optimal threshold τ_0 on channel norm is presented in Figure 9. The threshold values of vector τ_0 that maximize throughput η at corresponding SNR point are obtained from exhaustive search using (31). Figure 9 reveals that

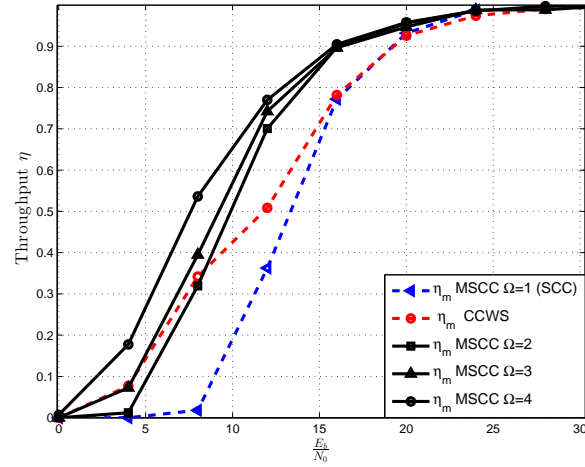


Fig. 10. Throughput comparison of SCC, CCWS and MSCC.

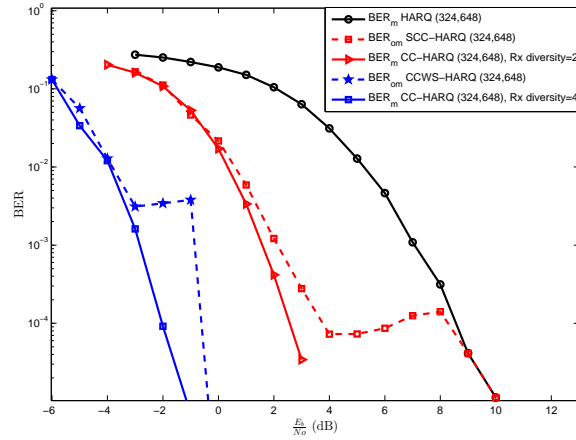


Fig. 11. BER comparison of ARQ, SCC, CCWS, CC Vs HARQ, SCC-HARQ, CCWS-HARQ and CC-HARQ at $\tau = 0$, $\tau = \tau_o$ and $\tau = \infty$ respectively using half rate LDPC code.

lower BER does not result into higher throughput. The BER performance corresponding to τ_∞ is lower than τ_0 . However, $\eta_o > \eta_\infty$. Figure 10 presents throughput gain of MSCC method over SCC (MSCC with single retransmission) and CCWS method. It is clear from Figure 10 that for two retransmissions ($\Omega = 2$), MSCC achieves significant throughput gain as compared to SCC. Throughput of MSCC with joint detection of three selective retransmissions is higher than CCWS

as well. As shown in Figure 10, there is throughput improvement by jointly detecting MSCC for three and four selective retransmissions. Throughput gain of MSCC method is significant in low and moderate SNR regime.

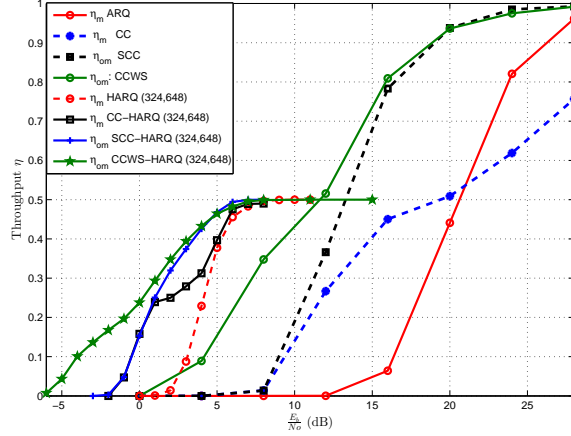


Fig. 12. Throughput comparison of ARQ, SCC, CCWS and CC Vs HARQ, SCC-HARQ, CCWS-HARQ and CC-HARQ at $\tau = 0$, $\tau = \tau_o$ and $\tau = \tau_f$, respectively using half rate LDPC code.

Figure 11 presents simulated BER performance of conventional retransmission methods and proposed selective retransmission methods with half-rate LDPC (648,324) code. We denote hybrid SCC and hybrid CCWS by SCC-HARQ and CCWS-HARQ, respectively in figure 11. Note that SCC-HARQ with threshold $\tau = 0$ is type-I HARQ, which is just single transmission without combining. The threshold $\tau = \infty$ represents full retransmission and conventional CC-HARQ. In figure 11, BER_{om} of SCC-HARQ represents BER of SCC-HARQ with optimal threshold τ_o on channel norm that maximizes throughput of the SISO-OFDM system. It is clear from figure 11 that in high SNR regime, BER_{om} for SCC-HARQ converges to BER_m of single transmission. The joint detection of CCWS-HARQ method with $\tau = \infty$ is equivalent to having four receive antennas. BER of CCWS-HARQ with optimal threshold τ_o is very close to that of with $\tau = \infty$.

Figure 12 provides Monte Carlo simulation results of throughput for ARQ, SCC, CCWS, HARQ, CC-HARQ, SCC-HARQ and CCWS-HARQ. As it is clear from figure 12 that in low SNR regime, throughput of conventional and proposed methods with FEC is higher than conventional and proposed methods without FEC. In high SNR regime, normalized throughput

of uncoded methods is higher than coded schemes. Figure 12 also reveals that throughput of CCWS has higher throughput as compared to SCC method. Also, proposed methods have higher throughput than conventional coded and uncoded CC methods. Thus, simulation results demonstrate efficacy of the proposed methods with and without channel coding.

VII. CONCLUSION

We presented performance analysis and throughput optimization of the proposed selective retransmission methods at modulation layer for OFDM system. We provide tight BER upper bound and lower throughput bound for SCC and CCWS methods. We optimize threshold τ to maximize throughput of the proposed selective Chase combining methods. We also generalize SCC method to multiple retransmission at modulation layer in response to packet failure. The simulation results show that there is marginal gap between throughput from Monte Carlo runs and that of from throughput analysis. The simulation results also demonstrate significant throughput gain of optimized selective retransmission methods over conventional Chase combining methods with and without channel coding.

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